

What Is Claimed Is:

- 1 1. A method for using a computer system to solve a global inequality
2 constrained optimization problem specified by a function f and a set of inequality
3 constraints $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f and p_i are scalar functions of a vector
4 $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the method comprising:
5 receiving a representation of the function f and the set of inequality
6 constraints at the computer system;
7 storing the representation in a memory within the computer system;
8 performing an interval inequality constrained global optimization process
9 to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
10 subject to the set of inequality constraints;
11 wherein performing the interval global optimization process involves,
12 applying term consistency to the set of inequality
13 constraints over a subbox \mathbf{X} , and
14 excluding any portion of the subbox \mathbf{X} that is proved to be
15 in violation of at least one member of the set of inequality
16 constraints.
- 1 2. The method of claim 1, further comprising:
2 linearizing the set of inequality constraints to produce a set of linear
3 inequality constraints with interval coefficients that enclose the nonlinear
4 constraints;
5 preconditioning the set of linear inequality constraints through additive
6 linear combinations to produce a preconditioned set of linear inequality
7 constraints;

8 applying term consistency to the set of preconditioned linear inequality
9 constraints over the subbox \mathbf{X} , and
10 excluding any portion of the subbox \mathbf{X} that violates any member of the set
11 of preconditioned linear inequality constraints.

1 3. The method of claim 2, further comprising:
2 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
3 point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and
4 including $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to
5 linearizing the set of inequality constraints.

1 4. The method of claim 2, further comprising removing from
2 consideration any inequality constraints that are not violated by more than a
3 specified amount for purposes of applying term consistency prior to linearizing
4 the set of inequality constraints.

1 5. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} ;
5 removing from consideration any subbox for which $f(\mathbf{x}) > f_bar$;
6 applying term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the
7 subbox \mathbf{X} ; and
8 excluding any portion of the subbox \mathbf{X} that violates the f_bar inequality.

1 6. The method of claim 1, wherein if the subbox \mathbf{X} is strictly feasible
2 ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval global optimization process
3 involves:
4 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
5 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
6 removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away
7 from zero, thereby indicating that the subbox does not include an extremum of
8 $f(\mathbf{x})$; and
9 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$
10 over the subbox \mathbf{X} ; and
11 excluding any portion of the subbox \mathbf{X} that violates any component of
12 $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

1 7. The method of claim 1, wherein if the subbox \mathbf{X} is strictly feasible
2 ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval global optimization process
3 involves:
4 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
5 function $f(\mathbf{x})$;
6 removing from consideration any subbox for which $H_{ii}(\mathbf{x})$ a diagonal
7 element of the Hessian over the subbox \mathbf{X} is always negative, indicating that the
8 function f is not convex over the subbox \mathbf{X} and consequently does not contain a
9 global minimum within the subbox \mathbf{X} ;
10 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
11 subbox \mathbf{X} ; and
12 excluding any portion of the subbox \mathbf{X} that violates a Hessian inequality.

1 8. The method of claim 1, wherein if the subbox \mathbf{X} is strictly feasible
2 ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval global optimization process
3 involves:
4 performing the Newton method, wherein performing the Newton method
5 involves,
6 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
7 function f evaluated with respect to a point \mathbf{x} over the subbox \mathbf{X} ,
8 computing an approximate inverse \mathbf{B} of the center of
9 $\mathbf{J}(\mathbf{x}, \mathbf{X})$,
10 using the approximate inverse \mathbf{B} to analytically determine
11 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
12 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
13 applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for
14 each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} ; and
15 excluding any portion of the subbox \mathbf{X} that violates a component.

1 9. The method of claim 1, wherein applying term consistency
2 involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$,
5 wherein the term $g(x'_j)$ can be analytically inverted to produce an inverse function
6 $g^{-1}(\mathbf{y})$;
7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(\mathbf{X}'_j) = h(\mathbf{X})$;
9 solving for $\mathbf{X}'_j = g^{-1}(h(\mathbf{X}))$; and
10 intersecting \mathbf{X}'_j with the j -th element of the subbox \mathbf{X} to produce a new
11 subbox \mathbf{X}^+ ;

12 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
13 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
14 the size of the subbox \mathbf{X} .

1 10. The method of claim 1, further comprising performing the Newton
2 method on the John conditions.

1 11. A computer-readable storage medium storing instructions that
2 when executed by a computer cause the computer to perform a method for using a
3 computer system to solve a global inequality constrained optimization problem
4 specified by a function f and a set of inequality constraints $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$),
5 wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the method
6 comprising:

7 receiving a representation of the function f and the set of inequality
8 constraints at the computer system;
9 storing the representation in a memory within the computer system;
10 performing an interval inequality constrained global optimization process
11 to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
12 subject to the set of inequality constraints;

13 wherein performing the interval global optimization process involves,
14 applying term consistency to the set of inequality
15 constraints over a subbox \mathbf{X} , and
16 excluding any portion of the subbox \mathbf{X} that is proved to be
17 in violation of at least one member of the set of inequality
18 constraints.

1 12. The computer-readable storage medium of claim 11, wherein the
2 method further comprises:
3 linearizing the set of inequality constraints to produce a set of linear
4 inequality constraints with interval coefficients that enclose the nonlinear
5 constraints;
6 preconditioning the set of linear inequality constraints through additive
7 linear combinations to produce a preconditioned set of linear inequality
8 constraints;
9 applying term consistency to the set of preconditioned linear inequality
10 constraints over the subbox **X**, and
11 excluding any portion of the subbox **X** that violates any member of the set
12 of preconditioned linear inequality constraints.

1 13. The computer-readable storage medium of claim 12, wherein the
2 method further comprises:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and
5 including $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to
6 linearizing the set of inequality constraints.

1 14. The computer-readable storage medium of claim 12, wherein the
2 method further comprises removing from consideration any inequality constraints
3 that are not violated by more than a specified amount for purposes of applying
4 term consistency prior to linearizing the set of inequality constraints.

1 15. The computer-readable storage medium of claim 11, wherein
2 performing the interval global optimization process involves:

1 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
2 point \mathbf{x} ;
3 removing from consideration any subbox for which $f(\mathbf{x}) > f_bar$;
4 applying term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the
5 subbox \mathbf{X} ; and
6 excluding any portion of the subbox \mathbf{X} that violates the f_bar inequality.

1 16. The computer-readable storage medium of claim 11, wherein if the
2 subbox \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval
3 global optimization process involves:
4 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
5 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
6 removing from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is bounded away
7 from zero, thereby indicating that the subbox does not include an extremum of
8 $f(\mathbf{x})$; and
9 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$
10 over the subbox \mathbf{X} ; and
11 excluding any portion of the subbox \mathbf{X} that violates any component of
12 $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

1 17. The computer-readable storage medium of claim 11, wherein if the
2 subbox \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval
3 global optimization process involves:
4 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
5 function $f(\mathbf{x})$;
6 removing from consideration any subbox for which $H_{ii}(\mathbf{x})$ a diagonal
7 element of the Hessian over the subbox \mathbf{X} is always negative, indicating that the

8 function f is not convex over the subbox \mathbf{X} and consequently does not contain a
 9 global minimum within the subbox \mathbf{X} ;
 10 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
 11 subbox \mathbf{X} ; and
 12 excluding any portion of the subbox \mathbf{X} that violates a Hessian inequality.

1 18. The computer-readable storage medium of claim 11, wherein if the
 2 subbox \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval
 3 global optimization process involves:

4 performing the Newton method, wherein performing the Newton method
 5 involves,

6 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
 7 function f evaluated with respect to a point \mathbf{x} over the subbox \mathbf{X} ,
 8 computing an approximate inverse \mathbf{B} of the center of
 9 $\mathbf{J}(\mathbf{x}, \mathbf{X})$,
 10 using the approximate inverse \mathbf{B} to analytically determine
 11 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
 12 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
 13 applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for
 14 each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} ; and
 15 excluding any portion of the subbox \mathbf{X} that violates a component.

1 19. The computer-readable storage medium of claim 11, wherein
 2 applying term consistency involves:
 3 symbolically manipulating an equation within the computer system to
 4 solve for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$,

5 wherein the term $g(x'_j)$ can be analytically inverted to produce an inverse function
 6 $g^{-1}(y)$;
 7 substituting the subbox \mathbf{X} into the modified equation to produce the
 8 equation $g(x'_j) = h(\mathbf{X})$;
 9 solving for $x'_j = g^{-1}(h(\mathbf{X}))$; and
 10 intersecting x'_j with the j -th element of the subbox \mathbf{X} to produce a new
 11 subbox \mathbf{X}^+ ;
 12 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
 13 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
 14 the size of the subbox \mathbf{X} .

1 20. The computer-readable storage medium of claim 11, wherein the
 2 method further comprises performing the Newton method on the John conditions.

1 21. An apparatus for using a computer system to solve a global
 2 inequality constrained optimization problem specified by a function f and a set of
 3 inequality constraints $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f is a scalar function of a
 4 vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the apparatus comprising:
 5 a receiving mechanism that is configured to receive a representation of the
 6 function f and the set of inequality constraints at the computer system;
 7 a memory within the computer system for storing the representation;
 8 a global optimizer that is configured to perform an interval inequality
 9 constrained global optimization process to compute guaranteed bounds on a
 10 globally minimum value of the function $f(\mathbf{x})$ subject to the set of inequality
 11 constraints;
 12 a term consistency mechanism within the global optimizer that is
 13 configured to,

14 apply term consistency to the set of inequality constraints
15 over a subbox \mathbf{X} , and to
16 exclude any portion of the subbox \mathbf{X} that is proved to be in
17 violation of at least one member of the set of inequality constraints.

1 22. The apparatus of claim 21, further comprising:
2 a linearizing mechanism that is configured to linearize the set of inequality
3 constraints to produce a set of linear inequality constraints with interval
4 coefficients that enclose the nonlinear constraints; and
5 a preconditioning mechanism that is configured to precondition the set of
6 linear inequality constraints through additive linear combinations to produce a
7 preconditioned set of linear inequality constraints;
8 wherein the term consistency mechanism is configured to,
9 apply term consistency to the set of preconditioned linear
10 inequality constraints over the subbox \mathbf{X} , and to
11 exclude any portion of the subbox \mathbf{X} that violates any
12 member of the set of preconditioned linear inequality constraints.

1 23. The apparatus of claim 22, wherein the global optimizer is
2 configured to:
3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and to
5 include $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to linearizing
6 the set of inequality constraints.

1 24. The apparatus of claim 22, wherein the term consistency
2 mechanism is configured to remove from consideration any inequality constraints

3 that are not violated by more than a specified amount for purposes of applying
4 term consistency prior to linearizing the set of inequality constraints.

1 25. The apparatus of claim 21,
2 wherein the global optimizer is configured to,
3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$
4 at a feasible point \mathbf{x} , and to
5 remove from consideration any subbox for which
6 $f(\mathbf{x}) > f_bar$;
7 wherein the term consistency mechanism is configured to,
8 apply term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$
9 over the subbox \mathbf{X} , and to
10 exclude any portion of the subbox \mathbf{X} that violates the f_bar
11 inequality.

1 26. The apparatus of claim 21, wherein if the subbox \mathbf{X} is strictly
2 feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to,
4 determine a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$
5 includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$), and to
6 remove from consideration any subbox for which $\mathbf{g}(\mathbf{x})$ is
7 bounded away from zero, thereby indicating that the subbox does
8 not include an extremum of $f(\mathbf{x})$; and
9 the term consistency mechanism is configured to,
10 apply term consistency to each component $g_i(\mathbf{x})=0$
11 ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} , and to

12 exclude any portion of the subbox **X** that violates any
13 component of $\mathbf{g}(\mathbf{x})=0$.

1 27. The apparatus of claim 21, wherein if the subbox **X** is strictly
2 feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to,
4 determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the
5 Hessian of the function $f(\mathbf{x})$, and to
6 remove from consideration any subbox for which $H_{ii}(\mathbf{x})$ a
7 diagonal element of the Hessian over the subbox **X** is always
8 negative, indicating that the function f is not convex over the
9 subbox **X** and consequently does not contain a global minimum
10 within the subbox **X**; and
11 the term consistency mechanism is configured to,
12 apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$
13 ($i=1, \dots, n$) over the subbox **X**, and to
14 exclude any portion of the subbox **X** that violates a Hessian
15 inequality.

1 28. The apparatus of claim 21, wherein if the subbox **X** is strictly
2 feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to perform the Newton method, wherein
4 performing the Newton method involves,
5 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
6 function f evaluated with respect to a point \mathbf{x} over the subbox **X**,
7 computing an approximate inverse \mathbf{B} of the center of
8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$, and

9 using the approximate inverse \mathbf{B} to analytically determine
10 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
11 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
12 the term consistency mechanism is configured to,
13 apply term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$
14 ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} , and to
15 exclude any portion of the subbox \mathbf{X} that violates a
16 component.

1 29. The apparatus of claim 21, wherein the term consistency
2 mechanism is configured to:
3 symbolically manipulate an equation within the computer system to solve
4 for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$, wherein
5 the term $g(x'_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;
6 substitute the subbox \mathbf{X} into the modified equation to produce the equation
7 $g(\mathbf{X}'_j) = h(\mathbf{X})$;
8 solve for $\mathbf{X}'_j = g^{-1}(h(\mathbf{X}))$; and
9 intersect \mathbf{X}'_j with the j -th element of the subbox \mathbf{X} to produce a new
10 subbox \mathbf{X}^+ ;
11 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
12 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
13 the size of the subbox \mathbf{X} .

1 30. The apparatus of claim 21, wherein the global optimizer is
2 configured to apply the Newton method to the John conditions.